Bit-Collapse Theory (BCT)
Born-Rule Constraints from Stable Records and Accessible Algebra

Phase-2 note, rewritten in canonical notation

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Abstract

This note presents a Born-rule route aligned with the Phase-2 framing of Bit-Collapse Theory (BCT). No modified quantum dynamics is introduced. Global Schrödinger evolution may remain unitary, while "collapse" is treated as an agent-relative class-label update tied to accessible stable records. Outcomes are represented as record-defined macro-conditions and associated projectors. A probability assignment over those projectors is then constrained by standard operational consistency requirements. Under those requirements, a Gleason-style uniqueness step yields the Born form. An appendix states the physical grounding of the constraints in the language of accessible algebra, stable records, coarse-grained exclusivity, and representation-independence.

1. Scope and canonical notation

This document is a constraint-and-uniqueness note. It does not propose a new collapse dynamics, a new Hamiltonian, or a departure from standard quantum predictions. Its narrower claim is the following: once outcomes are represented as stable record macro-conditions inside a fixed Phase-2 operational regime, the probability assignment is forced into the Born form by standard consistency requirements.

Canonical notation used throughout:
- Agent A: finite physical system with bounded access and finite integration.
- Accessible algebra AA(tau): observables physically accessible to A within the integration window tau.
- Persistence time tau_p: operational persistence / retention time of a record.
- Loss time tau_loss: characteristic decay timescale of a record buffer or trace.
- Threshold tau*: minimal persistence / retention threshold required for a record to count as event-like.
- Information threshold I_c: conservative record / which-way threshold (default: one effective bit).
- Global quantum state |Psi>: unitary state of the overall system, including system, apparatus, and environment.
- Experienced state [Psi]_A: equivalence-class label induced by AA(tau) and A-accessible stable records.

2. Outcomes as record-stable macro-conditions

Consider a system S coupled to an apparatus M and an environment E under standard unitary evolution. A measurement-like interaction yields an entangled state of the schematic form

$$|\Psi\rangle = \sum_i \alpha_i |i\rangle_S |M_i\rangle_M |E_i\rangle_E,$$

with approximately orthogonal environmental imprints at the coarse-graining relevant for records. In Phase 2, an outcome i is not treated as a primitive "measurement result," but as a record-defined macro-condition R_i. Operationally, R_i is a pointer-plus-imprint condition that:
(1) is accessible within AA(tau),

(2) persists long enough to satisfy tau_p >= tau* in the relevant record channel, and
(3) is stable enough to count as a record for A within the frozen operational regime.

Each macro-condition R_i corresponds to a macro-subspace H_R_i of the enlarged Hilbert space. Let Pi_i denote the projector onto H_R_i. At the adopted coarse-grained resolution, these record conditions are treated as mutually exclusive and exhaustive:

Pi_i Pi_j = 0 for i != j,
sum_i Pi_i = I.

This is not a metaphysical claim about exact microscopic separability. It is a statement about the coarse-graining under which stable records are defined.

3. The probability assignment

We seek a map mu(Pi | Psi) in [0,1], interpreted as the support that the global state |Psi> provides for the stable record macro-condition represented by projector Pi.

The constraints are the standard Phase-2 ones:
(C1) Non-negativity: mu(Pi | Psi) >= 0.
(C2) Normalization: mu(I | Psi) = 1.
(C3) Finite additivity on orthogonal projectors: if Pi Phi = 0, then mu(Pi + Phi | Psi) = mu(Pi | Psi) + mu(Phi | Psi).
(C4) Non-contextuality at fixed record condition: mu(Pi | Psi) depends only on Pi and |Psi>, not on which other compatible projectors appear beside Pi in a particular decomposition.
(C5) Unitary covariance: for any unitary U, mu(U Pi U^dagger | U|Psi>) = mu(Pi | Psi).

In Phase 2 these are not imported as free-floating metaphysics. They are the minimal consistency requirements for any support-assignment intended to track record-defined macro-conditions inside a frozen regime.

4. Gleason-style uniqueness and the Born form

For Hilbert spaces of dimension at least three, Gleason-style results imply that any non-negative, normalized, finitely additive, non-contextual assignment on projectors has the form

mu(Pi | Psi) = Tr(rho_Psi Pi),

for a unique positive trace-one operator rho_Psi associated with the preparation. For a pure preparation rho_Psi = |Psi><Psi|, this reduces to

mu(Pi | Psi) = <Psi| Pi |Psi>.

For one-dimensional idealized projectors the familiar Born weights |alpha_i|^2 are recovered.

Phase-2 policy for the qubit caveat:
Gleason's original theorem does not cover dimension two when one works only with bare projective measurements on an isolated qubit. Phase 2 therefore adopts the physically natural embedding policy: record projectors are macroscopic and live in the enlarged Hilbert space of system plus apparatus plus environment, where dimension at least three is generic. The uniqueness claim is about record projectors at that enlarged level, not about isolated qubit projectors taken in abstraction.

5. Why these constraints fit the Phase-2 record picture

The grounding question is straightforward: why should C1-C5 be accepted once outcomes are represented by stable records? The answer is operational rather than mystical.

Non-negativity and normalization are the minimal requirements for any support-assignment over an exhaustive family of record conditions. Additivity follows because orthogonal record projectors represent mutually exclusive macro-conditions at the chosen coarse-graining: the support for "R_i or R_j" must equal the sum of the supports when those conditions are disjoint. Non-contextuality is read narrowly: if two experimental descriptions correspond to the same record projector Pi_i, then the support for that record condition cannot depend on the bookkeeping context in which Pi_i appears. Unitary covariance is simply representation-independence: a joint change of basis in both state and projector cannot change the physical support of the same record condition.

The key point is that BCT does not ask the probability assignment to track micro-branch labels as such. It asks it to track record-defined macro-conditions. Once that move is made, the Born form is the unique consistent assignment under the stated operational constraints.

6. Relation to decoherence, Quantum Darwinism, and other Born programs

This note does not claim that decoherence by itself derives the Born rule. Decoherence and record proliferation enter earlier in the chain: they motivate why record projectors are the relevant coarse-grained events in the first place. Approximate orthogonality of environmental imprints is a structural feature of the recording chain, not yet a probability assignment.

Relative to other Born-rule programs, the Phase-2 stance is modest. Gleason provides the uniqueness theorem once the constraints are accepted. Deutsch-Wallace type approaches motivate similar constraints through rational consistency. Envariance emphasizes symmetry in system-environment correlations. BCT's distinctive claim is not new probability mathematics. Its distinctive claim is about what the projectors represent: stable, accessible, record-defined macro-conditions tied to experienced definiteness for an agent.

7. Conclusion

The Born-rule route in Phase 2 is deliberately narrow. It does not derive new dynamics, and it does not pretend to conjure probabilities from nothing. It says: if outcomes are represented as record-stable macro-conditions, and if the support-assignment over those macro-conditions obeys the standard consistency constraints appropriate to a frozen operational regime, then the Born form follows uniquely. BCT's added content lies in the ontological reading of those macro-conditions: they are not abstract branch labels, but stable record conditions that refine the agent-relative experienced state [Psi]_A.

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